

Name: _____

This quiz covers: Chapter 2.4 and 2.5.

DUE: Monday 2/20 at beginning of class.

Directions: Complete all questions and **show all applicable work**. Partial credit will be given. Please feel free to consult your text, notes and the online direction field solver as you see fit. Please do not discuss with other people (except the professor) or use the internet at large (including Wolfram Alpha). Each part is worth 5 points.

1.) (Chapter 2.4) Given the differential equation

$$t \frac{dy}{dx} + t^2 y = \sin(t)$$

with the initial condition $y(0) = 2$.

1. Without solving the differential equation, give an interval which the solution of the IVP is certain to exist.
2. Plot the direction field of the differential equation.
3. Write a closed form solution to the differential equation. An answer in terms of an integral is acceptable. See chapter 2.1.

2.) (Chapter 2.5) A private pond with no existing fish is to be stocked with the rare pi-fish. The landowner has budgeted \$1000 to spend on stock. You are to advise the landowner on how many pi-fish to stock such that she is not wasting money, but will **undoubtedly** have a successful breeding population. The pond has been determined to support up to 1400 pi-fish. The stocking fish come in two types:

- One-year-old fish at \$1.00 each. The minimum stocking necessary under normal conditions to create a self-sustaining population is 600 fish. The fish triple in population each year given sufficient food, space and a lack of predators.
- Two-year old fish at \$2.60 each. The minimum stocking necessary under normal conditions to create a self-sustaining population is 300 fish. The fish double in population each year given sufficient food, space and a lack of predators.

1. Provide a model for both types of fish.
2. Provide direction fields for each fish. Do they make sense?
3. What would you recommend to meet the goal and not risk a failure, assuming you can only choose one type and not a mixture of both? Since there may not be one definitive answer, clearly provide your rationale. This becomes your initial condition.
4. Assuming a solution is found to the DE paired with your given initial condition, is it a unique solution (mathematically speaking)? State why. (Do *not* solve it; see Chap. 2.4)
5. Discuss issues with the model and your solution in general. Eg.) Does the premise sound reasonable? Does your answer sound reasonable? What issues is the model overlooking or assuming? Is the budget realistic? Does this *undoubtedly* provide a breeding population? Is there a better way?