This quiz covers: Chapter 3. **DUE: 10:25 in my office or MAC 126**.

Directions: Complete all questions and **show all applicable work.** Partial credit will be given. You may use a calculator and one sheet of paper with handwritten notes. Please do not discuss with other people (except the professor) or use the Internet.

1.) [14pts] Assume that the solutions:

$$y_1 = 5e^{3t} \text{ and } y_2 = -2e^{3t}$$

solve a given 2nd order homogeneous DE. Do they form a fundamental set of solutions for the DE? Explain clearly why or why not.

2.) [20pts] Find the general solution to the following DE:

$$y'' - by' + 4y = 0.$$

for $b = 3$, $b = 4$, and $b = 5$. Your answer should consist of three solutions. For $b = 5$, additionally find the specific solution given initial conditions $y(0) = 0$ and $y'(0) = 1$. Please complete on separate paper.

3.) [20pts each] Using **BOTH** Variation of Parameters (formula or derivation) and the Method of Undetermined Coefficients find the general solution to the differential equation:

$$y'' + 4y' - 5y = e^{2t}.$$

Please complete on separate paper.

4.) An important part of a motorcycle's front suspension is the relationship between the spring (which holds the weight of the motorcycle) and the shock (which dampens spring oscillations created by road irregularities). Without a shock the motorcycle may lose control due to spring oscillations. On the other hand, if the shock is too stiff the rider and motorcycle will be damaged by vibrations produced by road irregularities.

A. [10pts] An engineering company is designing a new front shock and is attempting to select the proper shock dampening coefficient γ. Inside a testing chamber is a vertically aligned spring/shock with a 200kg mass attached. The mass compresses[1] the spring by 8cm. To simulate road unevenness and driving conditions the mass is externally acted upon by a force of $3\sin(\frac{t}{2})$ Newtons. Initially the spring is considered at rest. **Setup, but do not solve**, a differential equation to model this problem in terms of γ, including initial conditions.

B. [4pts each] Now assume that the differential equation above was solved for various choices of γ and the following solutions were to be considered: (Note: These are not actual solutions to the above DE.)

- $u = 4\sin(\frac{t}{2}) - 3\cos(\frac{t}{2})$
- $u = 7e^{-2t}\sin(\frac{t}{2}) - 12e^{-2t}\cos(\frac{t}{2})$
- $u = -4e^{2t} + 3e^{3t}$
- $u = 0$

For each of the above, describe the motion exhibited and if it is an ideal solution to the problem. That is, would the motion described by the solution be ideal to have on the front of a motorcycle, where u is the vertical displacement of the spring over time. It may be useful to plots these.

[1] Treat this the same as stretching. The difference between compressing and stretching a spring is mathematically is irrelevant.